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Critical behaviour of an m-vector spin glass for $m = \infty$

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Abstract. A special case of the spin glass model is considered in which the number of spin components m becomes infinitely large. We derive a field theory Hamiltonian for this model and show that its upper critical dimensionality is eight. The critical exponents for this theory are calculated in an expansion in $\varepsilon = 8 - d$ to second order. We notice that this second-order expansion is identical to that for the $p \to \infty$ limit of the Q^3 model in $\varepsilon = 6 - d$.

1. Introduction

There has recently been interest in systems whose effective upper critical dimensionality (UCD) is eight and their relation to theories with an upper critical dimension of six. The two previous examples are the lattice animal problem (Lubensky and Isaacson 1979) and the field theory model of Anderson localisation proposed by Harris and Lubensky (1981). These two models have been shown to be equivalent to one another (Lubensky and McKane 1981) and to the Yang–Lee edge singularity near six dimensions (Parisi and Sourlas 1981). In this paper we consider the usual model of a spin glass with Hamiltonian

$$H = -\tfrac{1}{2} \sum_{i,j,a} J_{ij} S_{ia} S_{ja}, \qquad (1.1)$$

where S_{ia} is a cartesian component of an m-component spin at site i of a hypercubic lattice and J_{ij} is a random coupling between nearest neighbour spins, in the limit when the number of components m tends to infinity. Surprisingly it turns out that the UCD of the large-m limit is eight and not six as it is for any finite m. This curious result is the subject of this paper.

In the rest of this section we will show how the field theory Hamiltonian is derived in the large-m limit. In § 2 we see why the UCD is eight and also calculate the exponents for the theory to second order in $\varepsilon = 8 - d$. In § 3 we notice that these exponents are the same as the $p = \infty$ limit of those in an expansion about six dimensions for the Q^3 tensor field theory described by Priest and Lubensky (1976). In this theory Q is a p-dimensional traceless symmetric tensor and we consider the case in which the coupling constant is imaginary. One might therefore hope to prove similar identities to those between the lattice animal and Yang–Lee edge singularity theories.

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We return to the Hamiltonian (1.1) and consider the partition function of the system replicated n times

$$Z^n(J) = \mathrm{Tr}_s \exp\Big(-\beta \sum_{\substack{i,j,a\\ \eta}} J_{ij} S^\eta_{ia} S^\eta_{ja}\Big) \tag{1.2}$$

$$\equiv \mathrm{Tr}_s \exp(-\beta H^{(n)}) \tag{1.3}$$

where each replica is labelled by η and η runs from 1 to n. Tr_s denotes a trace over all spin variables. We define the average over all possible configurations of the coupling J_{ij} by

$$\langle Z^n \rangle_J = \int \mathrm{d}[J]\, P[J]\, \mathrm{Tr}_s \exp(-\beta H^{(n)}) \tag{1.4}$$

where $P[J]$ is the probability distribution function for J. If we define the effective Hamiltonian for the theory by

$$\langle Z^n \rangle_J = \mathrm{Tr}_s \exp(-\beta \tilde{H}^{(n)}) \tag{1.5}$$

then we can write down $\tilde{H}^{(n)}$ formally by integrating (1.4) over J to give

$$\beta \tilde{H}^{(n)} = -\sum_{k=1}^{\infty} \sum_{i,j} C^{(k)} \Big(\beta \sum_{\eta=1}^{n} S^\eta_{ia} S^\eta_{ja}\Big)^k \tag{1.6}$$

where $C^{(k)}$ is the kth cumulant of the distribution $P[J]$. For the spin glass case $C^{(1)} = 0$ and if we further take $P[J]$ to be a gaussian distribution then $C^{(k)} = 0$, $k > 2$. Hence we have only one term remaining in $\tilde{H}^{(n)}$ and

$$\langle Z^n \rangle_J = \mathrm{Tr}_s \exp\Big(\tfrac{1}{4}\beta^2 \sum_{\substack{i,j,a,b\\ \eta,\nu}} K_{ij} S^\eta_{ja} S^\eta_{ia} S^\nu_{jb} S^\nu_{ib}\Big) \tag{1.7}$$

where $K_{ij} = 1$ if i, j are nearest neighbour sites and zero otherwise.

The application of the Hubbard–Stratonovich transformation (Hubbard 1972) gives

$$\langle Z^n \rangle_J = \int \mathscr{D} Q^{\eta\nu}_{ab} \exp\Big(-\frac{1}{\beta^2} \sum_{\substack{a,b,\eta,\nu\\ i,j}} (K^{-1})_{ij} Q^{\eta\nu}_{iab} Q^{\eta\nu}_{jab} + \sum_i \ln z_i\Big) \tag{1.8}$$

where

$$z_i = \mathrm{Tr}_s \exp \sum_{\substack{a,b\\ \eta,\nu}} Q^{\eta\nu}_{iab} S^\eta_{ia} S^\nu_{ib}. \tag{1.9}$$

We expand $\ln z_i$ to 3rd order in the Q fields, since in the region of interest near eight dimensions these terms dominate over those of higher order. If we choose the length of our spin to be m, i.e.

$$\langle S^\eta_a S^\nu_b \rangle = \delta_{ab}\, \delta^{\eta\nu}, \tag{1.10}$$

then we can show that the expansion of $\ln z$ to the third order in Q can be written

$$\int \mathrm{d}^d \boldsymbol{x} \Big(\frac{m}{m+2} \sum_{\substack{a,b\\ \eta}} (T^\eta_{ab})^2 + \sum_{\substack{a,b\\ \eta,\nu}} (Q^{\eta\nu}_{ab})^2 + \frac{8m^2}{3!(m+2)(m+4)} \sum_{\substack{a,b,c\\ \eta}} T^\eta_{ab} T^\eta_{bc} T^\eta_{ca}$$
$$+ \frac{8}{3!} \sum_{\substack{a,b,c\\ \eta,\nu,\lambda}} Q^{\eta\nu}_{ab} Q^{\nu\lambda}_{bc} Q^{\lambda\eta}_{ca} + \frac{1}{2!} \frac{m}{m+2}\, 8 \sum_{\substack{a,b,c\\ \eta,\nu}} T^\eta_{ab} Q^{\eta\nu}_{ac} Q^{\eta\nu}_{bc}\Big) \tag{1.11}$$

where we have separated our Q field into the diagonal part $Q_{ab}^{\alpha\alpha}=T_{ab}^{\alpha}$ and the off-diagonal part $Q_{ab}^{\eta\nu}=Q_{ab}^{\eta\nu}$ if $\eta\neq\nu$ and also taken the continuum limit (i.e. $\Sigma_i A_i \to \int \mathrm{d}^d \boldsymbol{x}\, A(\boldsymbol{x})$). This analysis is similar to that of Harris *et al* (1976) and Chen and Lubensky (1977) except that we have calculated the m dependence explicitly and retained the diagonal terms. T_{ab}^{η} is a traceless symmetric tensor which we call the quadrupole field, and the off-diagonal $Q_{ab}^{\eta\nu}$ field of equation (1.11) is the spin glass field of Harris *et al* (1976). There is another contribution to the effective Hamiltonian from the $\beta^{-2}\Sigma(K^{-1})_{ij}Q_{iab}^{\eta\nu}Q_{jab}^{\eta\nu}$ term in equation (1.8). To evaluate this we make a Fourier transformation into momentum space and then expand the resulting $K(\boldsymbol{q})$ as a power series in $\boldsymbol{q}$. If we then invert the transformation in terms of a continuous space variable $\boldsymbol{x}$ we have to lowest order in the derivatives

$$\int \mathrm{d}^d\boldsymbol{x}\, \beta^{-2}[(\nabla Q_{ab}^{\eta\nu})^2+(\nabla T_{ab}^{\eta})^2+\tilde{r}(Q_{ab}^{\eta\nu})^2+\tilde{r}(T_{ab}^{\eta})^2] \tag{1.12}$$

where $\tilde{r}$ is a constant which depends on the normalisation choice for the gaussian distribution. Thus, adding (1.11) and (1.12), the coefficient of $(T_{ab}^{\eta})^2$ is $\tilde{r}/\beta^2+m/(m+2)$ and the coefficient of $(Q_{ab}^{\eta\nu})^2$ is $\tilde{r}/\beta^2+1$, so the two fields have a difference in mass of order $1/m$. It is this difference in mass which in the $m\to\infty$ limit shifts the UCD from the six we would expect for a trilinear theory with finite m to eight. When m is very large but still finite, we expect that there will be a crossover at some temperature near the critical temperature from the infinite-m behaviour to the finite-m behaviour. This mechanism for boosting the UCD does not seem to have been encountered previously. The continuum field theory is now obtained by adding (1.11) and (1.12) and scaling the fields so that the coefficient of $(\nabla Q_{ab}^{\eta\nu})^2$ is $\frac{1}{4}$; the result is

$$\begin{aligned} H=\int \mathrm{d}^d\boldsymbol{x}\Big(&\tfrac{1}{4}\sum_{\substack{a,b\\ \eta,\nu}}[(\nabla Q_{ab}^{\eta\nu})^2+r(Q_{ab}^{\eta\nu})^2]+\tfrac{1}{4}\sum_{\substack{a,b\\ \eta}}[(\nabla T_{ab}^{\eta})^2+(r+\tau/m)(T_{ab}^{\eta})^2] \\ &+\frac{\omega}{3!}\sum_{\substack{a,b,c\\ \eta,\nu,\lambda}} Q_{ab}^{\eta\nu}Q_{bc}^{\nu\lambda}Q_{ca}^{\lambda\eta}+\frac{\omega m^2}{3!(m+2)(m+4)}\sum_{\substack{a,b,c\\ \eta}} T_{ab}^{\eta}T_{bc}^{\eta}T_{ca}^{\eta} \\ &+\frac{\omega m}{2!(m+2)}\sum_{\substack{a,b,c\\ \eta,\nu}} T_{ab}^{\eta}Q_{ac}^{\eta\nu}Q_{bc}^{\eta\nu}\Big) \end{aligned} \tag{1.13}$$

where ω, r and $r+\tau/m$ are the coefficients of $\boldsymbol{Q}^3$, $\boldsymbol{Q}^2$ and $\boldsymbol{T}^2$ which arise after we have rescaled the fields.

2. ε expansion

In this section we describe how the critical exponents were calculated in an ε expansion. We begin by expanding the coefficients of the trilinear terms in the Hamiltonian as a series in $1/m$. In the large-m limit it turns out that we can neglect all except the zeroth-order term and our Hamiltonian density becomes

$$\begin{aligned}\mathscr{H}=&\tfrac{1}{4}[(\nabla\boldsymbol{Q})^2+r\boldsymbol{Q}^2]+\tfrac{1}{4}[(\nabla\boldsymbol{T})^2+(r+\tau/m)\boldsymbol{T}^2]\\ &+\omega\boldsymbol{Q}^3/3!+\omega\boldsymbol{T}^3/3!+\omega\boldsymbol{T}\boldsymbol{Q}^2/2!\end{aligned} \tag{2.1}$$

where we have introduced a compact notation $Q_{ab}^{\eta\nu}=\boldsymbol{Q}$, $T_{ab}^{\eta}=\boldsymbol{T}$ and the relevant sums over indices can be deduced from (1.13). We cannot take the limit $m\to\infty$ before we renormalise the theory, as we generate terms of $\mathrm{O}(m)$ in the calculation, which give finite relevant contributions when multiplied by the τ/m term in the mass. We have checked for instance that the $1/m$ terms from the three-point interactions do not give non-zero contributions in the $m\to\infty$ limit. In fact, our $m\to\infty$ limit is like the $n\to 0$ limit which is taken in order to obtain percolation theory in terms of the $(n+1)$-state Potts model.

We can renormalise the two fields using the same renormalisation since they are essentially different parts of the same field. We define $\boldsymbol{Q}_{\mathrm{R}}=Z^{-1/2}\boldsymbol{Q}$, $\boldsymbol{T}_{\mathrm{R}}=Z^{-1/2}\boldsymbol{T}$, a renormalised mass m_{R}^2 and a renormalised coupling ω_{R}; then writing $\mathscr{H}$ in terms of renormalised fields and couplings

$$\begin{aligned}\mathscr{H}=&\tfrac{1}{4}[(\nabla\boldsymbol{T}_{\mathrm{R}})^2+(m_{\mathrm{R}}^2+\tau/m)\boldsymbol{T}_{\mathrm{R}}^2]+\tfrac{1}{4}[(\nabla\boldsymbol{Q}_{\mathrm{R}})^2+m_{\mathrm{R}}^2\boldsymbol{Q}_{\mathrm{R}}^2]\\&+(\omega_{\mathrm{R}}\mu^{\varepsilon/2-1}/3!)(\boldsymbol{T}_{\mathrm{R}}^3+\boldsymbol{Q}_{\mathrm{R}}^3+3\boldsymbol{T}_{\mathrm{R}}\boldsymbol{Q}_{\mathrm{R}}^2)+\tfrac{1}{4}(Z_Q-1)m_{\mathrm{R}}^2\boldsymbol{Q}_{\mathrm{R}}^2\\&+\tfrac{1}{4}(Z_T-1)m_{\mathrm{R}}^2\boldsymbol{T}_{\mathrm{R}}^2+\tfrac{1}{4}(Z-1)[(\nabla\boldsymbol{Q}_{\mathrm{R}})^2+(\nabla\boldsymbol{T}_{\mathrm{R}})^2]\\&+(1/3!)(\omega_0Z^{3/2}-\omega_{\mathrm{R}})\mu^{\varepsilon/2-1}(\boldsymbol{Q}_{\mathrm{R}}^3+\boldsymbol{T}_{\mathrm{R}}^3+3\boldsymbol{T}_{\mathrm{R}}\boldsymbol{Q}_{\mathrm{R}}^2)\end{aligned}\tag{2.2}$$

where

$$Z_Q=\frac{r}{m_{\mathrm{R}}^2}Z,\qquad Z_T=\left((Z-1)\frac{\tau}{m}\frac{1}{m_{\mathrm{R}}^2}+\frac{rZ}{m_{\mathrm{R}}^2}\right),$$

$\omega_0=\omega\mu^{1-\varepsilon/2}$, $\varepsilon=8-d$ and μ is a momentum scale.

We shall use the method of minimal subtraction ('t Hooft and Veltman 1972) of ε poles to calculate our ε expansion, since within this scheme the results are independent of the choice of external momenta for the vertex, and we have to calculate only the divergent parts of Feynman diagrams. Consider the calculation of the three-point vertex function in the Q (spin glass) field

$$\Gamma_{\mathrm{R}}^{(3,0)}(q^2,m_{\mathrm{R}}^2,\tau)=\omega_{\mathrm{R}}\mu^{\varepsilon/2-1}+(\omega_0Z^{3/2}-\omega_{\mathrm{R}})\mu^{\varepsilon/2-1}+A\tag{2.3}$$

where A is the sum of all three-point diagrams. At first order there are two diagrams, shown in figure 1(a), which give the following contribution to A:

$$(-3m+1)\omega_{\mathrm{R}}^3\mu^{3\varepsilon/2-3}\int_p\frac{1}{(p^2+m_{\mathrm{R}}^2)^3}+3m\omega_{\mathrm{R}}^3\mu^{3\varepsilon/2-3}\int_p\frac{1}{(p^2+m_{\mathrm{R}}^2)^2(p^2+m_{\mathrm{R}}^2+\tau/m)}\tag{2.4}$$

where $\int_p$ denotes $\int\mathrm{d}^dp/(2\pi)^d$ and we have chosen the external momenta to be zero. Adding the two parts together and substituting for A in (2.3),

$$\begin{aligned}\Gamma_{\mathrm{R}}^{(3,0)}(0,m_{\mathrm{R}}^2,\tau)=&\,\omega_{\mathrm{R}}\mu^{\varepsilon/2-1}+(\omega_0Z^{3/2}-\omega_{\mathrm{R}})\mu^{\varepsilon/2-1}\\&+\omega_{\mathrm{R}}^3\mu^{3\varepsilon/2-3}\left(\int_p\frac{1}{(p^2+m_{\mathrm{R}}^2)^3}-3\tau\int_p\frac{1}{(p^2+m_{\mathrm{R}}^2)^4}+\text{convergent terms}\right)\\&+\mathrm{O}(\omega_{\mathrm{R}}^5).\end{aligned}\tag{2.5}$$

In the minimal subtraction scheme the convergent terms do not contribute. We are left with two integrals, the first of which becomes marginal near six dimensions and the second near eight dimensions; hence near eight dimensions we need only include the contributions from the second integral. $\Gamma_{\mathrm{R}}^{(3,0)}$ is finite by construction and

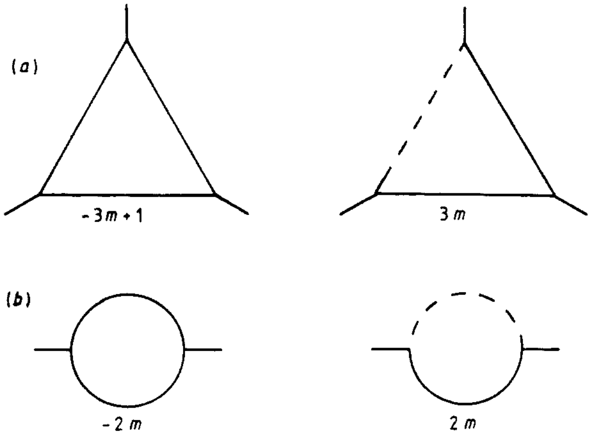

Figure 1. Feynman diagrams which contribute to (*a*) $\Gamma_{\mathrm{R}}^{(3,0)}(q^2, m_{\mathrm{R}}^2, \tau)$ and (*b*) $\Gamma_{\mathrm{R}}^{(2,0)}(q^2, m_{\mathrm{R}}^2, \tau)$ at one loop. Full lines denote propagators of the spin glass (Q) field and broken lines those of the quadrupole (T) field. The numbers shown under each diagram are the overall symmetry factors in the limit that n the number of replicas tends to zero.

therefore the counterterm $(\omega_0 Z^{3/2} - \omega_{\mathrm{R}})$ must exactly cancel the ε poles from A; hence

$$(\omega_0 Z^{3/2} - \omega_{\mathrm{R}})\mu^{\varepsilon/2-1} = (3\omega_{\mathrm{R}}^3/\varepsilon)\tau\mu^{3\varepsilon/2-3} m_{\mathrm{R}}^{-\varepsilon} + \mathrm{O}(\omega_{\mathrm{R}}^5) \tag{2.6}$$

where we have absorbed a factor of $S_d/(2\pi)^d$ into the coupling constant as usual. We can derive the same equation by considering either of the two other three-point vertices, and this was done to check the result.

The wavefunction renormalisation Z can be calculated from either the two-point spin glass or quadrupole vertex function. We shall consider the spin glass function:

$$\Gamma_{\mathrm{R}}^{(2,0)}(q^2, m_{\mathrm{R}}^2 = 0, \tau) = q^2 + (Z-1)q^2 - \Sigma_Q(q^2, m_{\mathrm{R}}^2 = 0, \tau) \tag{2.7}$$

where Σ_Q is the sum of all Feynman diagrams with two external spin glass legs. In order to simplify the calculation of the two-loop diagrams we calculate the derivative $\Gamma_{\mathrm{R}}^{(2,0)}$ with respect to q^2. Since $\Gamma_{\mathrm{R}}^{(2,0)}$ is finite by construction we have the equation

$$Z - 1 = \partial(\Sigma_Q)/\partial q^2. \tag{2.8}$$

At first order the graphs which contribute to Σ_Q are shown in figure 1(*b*). Writing down the values of these graphs

$$\begin{aligned}\Sigma_Q &= \left(-\int_p \frac{1}{p^2(p-q)^2} + \int_p \frac{1}{(p^2+\tau/m)(p-q)^2}\right)\omega_{\mathrm{R}}^2\mu^{\varepsilon-1}2m \\ &= -2\tau\int_p \frac{1}{p^4(p-q)^2} + \frac{\tau^2}{m}\int_p \frac{1}{p^6(p-q)^2} + \text{convergent terms}.\end{aligned} \tag{2.9}$$

When we differentiate with respect to q^2 the second term becomes convergent and so the only contribution is from the first integral. We find

$$Z = 1 + (\tau/\varepsilon)\omega_{\mathrm{R}}^2\mu^{-2} + \mathrm{O}(\omega_{\mathrm{R}}^4). \tag{2.10}$$

If we substitute this in equation (2.6) we find the following equation for ω_0 in terms of ω_{R}:

$$\omega_0 = \omega_{\mathrm{R}} + (3/2\varepsilon)\omega_{\mathrm{R}}^3\tau\mu^{-2} + \mathrm{O}(\omega_{\mathrm{R}}^5). \tag{2.11}$$

In order to find a fixed point and calculate exponents, we need to choose an effective coupling related to our original coupling which has marginal dimensions $[p]^{\varepsilon}$. This is exactly the same as in the calculation of the ε expansion for lattice animals (Lubensky and Isaacson 1979). We choose

$$g_0 = \omega_0^2 \tau \mu^{-2}, \qquad g_R = \omega_R^2 \tau \mu^{-2}, \tag{2.12}$$

to define our new coupling. We obtain the equation for g_0 in terms of g_R by squaring equation (2.11):

$$g_0 = g_R + 3g_R^2/\varepsilon + O(g_R^3). \tag{2.13}$$

The β function for the coupling constant g_R is given by

$$\beta(g_R) = \mu \frac{\partial g_R}{\partial \mu} = -\varepsilon(\partial \ln g_0/\partial g_R)^{-1} \tag{2.14}$$

so from (2.13)

$$\beta(g_R) = -\varepsilon g_R + 3g_R^2 + O(g_R^3) \tag{2.15}$$

and the fixed point value $\beta(g^*) = 0$ is given by

$$g^* = \varepsilon/3. \tag{2.16}$$

The critical exponent η is given as usual by

$$\eta = \left(\mu \frac{\partial \ln Z}{\partial \mu}\right)\bigg|_{g_R = g^*} = \left(\beta(g_R)\frac{\partial \ln Z}{\partial g_R}\right)\bigg|_{g_R = g^*} \tag{2.17}$$

and we find

$$\eta = -\varepsilon/3. \tag{2.18}$$

The calculation of the second exponent involves the evaluation of Z_T and Z_Q. We consider the two-point vertex function or spin glass fields in the massive, zero external momentum case:

$$\Gamma_R^{(2,0)}(q^2 = 0, m_R^2, \tau) = m_R^2 + m_R^2(Z_Q - 1) - \Sigma_Q(q^2 = 0, m_R^2, \tau). \tag{2.19}$$

At first order

$$\Sigma_Q(q^2 = 0, m_R^2, \tau) = -2\omega_R^2 \mu^{\varepsilon - 2} \tau \left(\int_p \frac{1}{(p^2 + m_R^2)^3} - \int_p \frac{\tau/m}{(p^2 + m_R^2)^4} \right). \tag{2.20}$$

The first term is proportional to m_R^2 and is the relevant one at this order. The second term is of $O(1/m)$ and so vanishes in the $m \to \infty$ limit. However, when we include the counterterm at second order this contribution is multiplied by m and becomes relevant. If instead of $\Gamma_R^{(2,0)}$ we consider $\Gamma_R^{(0,2)}$ and calculate Z_T, the m_R^2 parts are identical and it is only the $O(1/m)$ term that differs. The exponent $\nu^{-1} - 2 + \eta$ is therefore the same for both fields. If we define Z_{mass} to be the part of Z_Q and Z_T proportional to m_R^2 then

$$\nu^{-1} - 2 + \eta = \left(\beta(g_R)\frac{\partial \ln Z_{\text{mass}}}{\partial g_R}\right)\bigg|_{g_R = g^*} \tag{2.21}$$

and we find

$$\nu^{-1} - 2 + \eta = -2\varepsilon. \tag{2.22}$$

The calculation of second order is just a direct extension of the above. When we add together graphs of the same skeleton structure we find the expected cancellations to give us terms of $O(\tau^2 \omega_R^4)$ and additional less divergent terms. These less divergent terms are of O(1) as at first order, so that there is no problem in the $m \to \infty$ limit. We used the skeleton technique described in Vladimirov (1979) and de Alcantara Bonfim *et al* (1981) to simplify the evaluation of the integrals. At second order there are independent checks in the theory; we can check first that the vertices are independent of the choice of external momenta as predicted by the minimal subtraction scheme. Also we have 't Hooft (1973) identities which predict that the function $\beta(g_R)$ contains no ε poles. Our results to second order are

$$\begin{aligned} \beta(g_R) &= -g_R[\varepsilon - 3g_R + \tfrac{5}{2}g_R^2 + O(g_R^3)], \\ g^* &= \tfrac{1}{3}\varepsilon + \tfrac{5}{54}\varepsilon^2 + O(\varepsilon^3), \\ \eta &= -\tfrac{1}{3}\varepsilon - \tfrac{2}{27}\varepsilon^2 + O(\varepsilon^3), \\ \nu^{-1} - 2 + \eta &= -2\varepsilon + \tfrac{23}{18}\varepsilon^2 + O(\varepsilon^3). \end{aligned} \tag{2.23}$$

3. Conclusions

We begin by noticing that the ε expansions for exponents that we have calculated are identical to second order to those of the Q^3 model considered by Priest and Lubensky (1976). Their model is a tensor trilinear theory with Hamiltonian

$$\mathcal{H} = \tfrac{1}{4}\sum_{i,j} (\nabla Q_{ij})^2 + \tfrac{1}{4}\sum_{i,j} r(Q_{ij})^2 + \frac{u}{3!}\sum_{i,j,k} Q_{ij}Q_{jk}Q_{ki} \tag{3.1}$$

where $i, j = 1, 2, \ldots, p$ and Q_{ij} is a symmetric traceless tensor. We are interested in the special case $p \to \infty$, but for $p > 4$ the fixed point of the Q^3 theory is unphysical near six dimensions for real u. We therefore consider the case when u is pure imaginary and the theory is well defined in the $p \to \infty$ limit. We find that our exponents to second order in an expansion about eight dimensions for the $m \to \infty$ spin glass are identical to the second-order expansion of exponents of the above theory in the $p \to \infty$, imaginary coupling constant case. It seems likely that the exponents are the same to all orders in ε and there is a $d \to d-2$ rule similar to that between lattice animals and the Yang–Lee edge singularity. This means that we expect the critical behaviour of our model near d dimensions to be identical to that of the Q^3 model of Priest and Lubensky (1976) in $d-2$ dimensions. We have therefore calculated the $O(\varepsilon^3)$ contributions for the Q^3 model using the general results for exponents from de Alcantara Bonfim *et al* (1981). We find

$$\eta = -\tfrac{1}{3}\varepsilon - \tfrac{2}{27}\varepsilon^2 + [19\varepsilon^3/(2^2 \times 3^5)] + O(\varepsilon^4), \tag{3.2}$$

$$\nu^{-1} - 2 + \eta = -2\varepsilon + \tfrac{23}{18}\varepsilon^2 - [1087\varepsilon^3/(2^3 \times 3^4)] + O(\varepsilon^4). \tag{3.3}$$

We have not extended the expansion for the spin glass model to the same order, but content ourselves with the speculation that by the methods used by Parisi and Sourlas (1981) it might be possible to prove the equivalence to all orders.

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